

Whitebeard's Treasure

Whitebeard, the notorious pirate of the West Bay, buried treasure on Tiki Island over 200 years ago. Archeologists recently discovered a map showing the location of the treasure. The location has generated quite a bit of media attention, much to the dismay of the archeologists. In order to allow both the media and archeologists to work together, officials have decided to erect two fences around the location, allowing the media access to the site, yet allowing the archeologists room to work. One fence encloses the actual area where the archeologists will work. Another fence surrounds the enclosed dig area.

Descriptions of the fencing locations have been provided to the media so they may indicate accessible areas for their employees. Use the given information to draw and label a quadrilateral on graph paper indicating the location of the two fences.

1. Corners of the first fence are located at points $A(11,3)$, $B(3,-11)$, $C(-13,-9)$ and $D(-5,9)$. The media must stay within this fenced area. Connect the points in alphabetical order, and then join point D to Point A.
2. Find and label the midpoints of each segment of quadrilateral ABCD, showing all work. Label the midpoints of the segments as follows:

\overline{AB} has midpoint Q,

\overline{BC} has midpoint R,

\overline{CD} has midpoint S,

\overline{DA} has midpoint T.

3. Connect the four midpoints in alphabetical order to create a new quadrilateral QRST. This quadrilateral represents the fence surrounding the archeological dig site.
4. Quadrilateral ABCD was an ordinary quadrilateral, but QRST is a special one. Determine the special name for quadrilateral QRST, and justify your answer using coordinate geometry in two different ways.

Notes

Materials:

One graphing calculator per student

Geometry TEKS Focus:

(G.3) **Geometric structure.** The student applies logical reasoning to justify and prove mathematical statements.

The student is expected to:

(B) construct and justify statements about geometric figures and their properties;

(G.7) Dimensionality and the geometry of location.

The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.

(B) use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons; and

Additional Geometry TEKS:

(G.3) **Geometric structure.** The student applies logical reasoning to justify and prove mathematical statements.

The student is expected to:

(D) use inductive reasoning to formulate a conjecture; and

(E) use deductive reasoning to prove a statement.

(G.7) Dimensionality and the geometry of location.

The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.

Teacher Notes

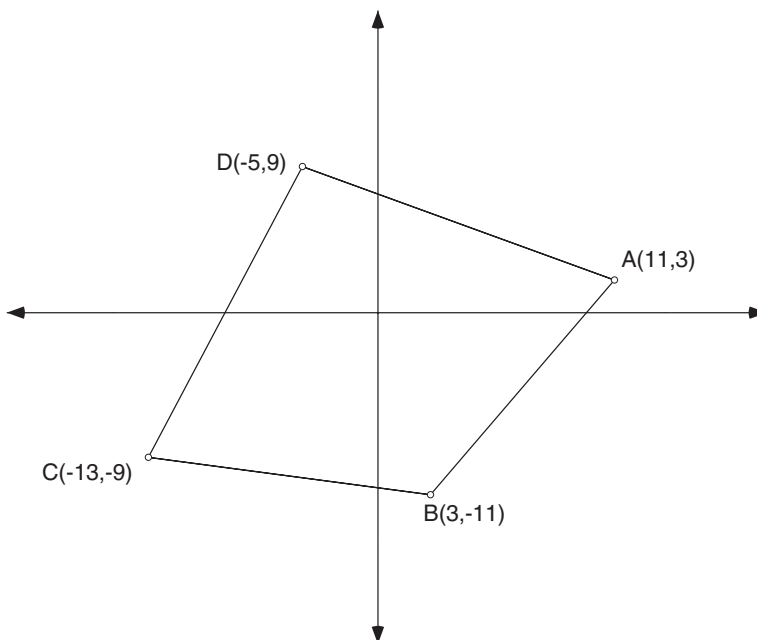
This performance task addresses the same mathematical concepts as the Wearable Art task later in this chapter. Whitebeard's Treasure gives the numerical coordinates. In Wearable Art the coordinates are given and the student must represent the situation using variable coordinates for the points. The teacher may choose to use one or both of these problems.

Scaffolding Questions:

- What is the formula for finding the midpoint of a line segment?
- Which of the quadrilaterals are special quadrilaterals?
- What are the characteristics of each special quadrilateral?
- What characteristics does quadrilateral QRST appear to possess that matches one of the special quadrilaterals?
- How can you prove these special characteristics?

Sample Solutions:

Quadrilateral ABCD is graphed as shown. This is the outer fence.



To find the midpoint of each segment of quadrilateral ABCD, use the midpoint formula.

The midpoint of the segment with endpoints (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$.

To find the midpoint of each segment, substitute the x and y values from the endpoints of the segment into the formula as follows:

Midpoint of \overline{AB} (Point Q)

$$\left(\frac{11 + 3}{2}, \frac{3 + (-11)}{2}\right) = (7, -4)$$

Midpoint of \overline{BC} (Point R)

$$\left(\frac{3 + (-13)}{2}, \frac{-11 + (-9)}{2}\right) = (-5, -10)$$

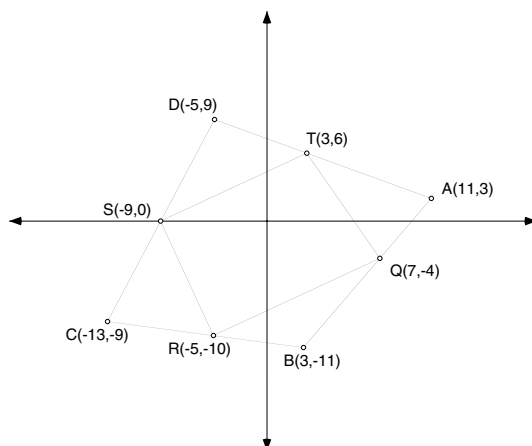
Midpoint of \overline{CD} (Point S)

$$\left(\frac{-13 + (-5)}{2}, \frac{-9 + 9}{2}\right) = (-9, 0)$$

Midpoint of \overline{DA} (Point T)

$$\left(\frac{-5 + 11}{2}, \frac{9 + 3}{2}\right) = (3, 6)$$

Graph the midpoints and connect them in alphabetical order to form a new quadrilateral QRST.



The student is expected to:

(A) use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures;

(C) derive and use formulas involving length, slope, and midpoint.

Connections to TAKS:

Objective 3: The student will demonstrate an understanding of linear functions.

Objective 7: The student will demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.

One teacher says . . .

“I had the students get out their notes before they started. I gave them two minutes to read the problem and think of questions they wanted to ask the class or me. Some of the scaffolding questions I asked were:

What types of quadrilaterals are there?

How do you know what type of quadrilateral it is?

How do you know when two lines are parallel?

How do you know when two lines are perpendicular?”

Quadrilateral QRST would be the fence that encloses the archeologists' dig site.

Quadrilateral QRST appears to be a parallelogram because the opposite sides of the newly formed quadrilateral appear to be parallel. One way to prove that a quadrilateral is a parallelogram is to prove that both pairs of opposite sides are parallel. Lines that have the same slope are parallel lines.

Use the slope formula:

The slope of the line through points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

The slope of \overleftrightarrow{RS} is $\frac{0 - (-10)}{-9 - (-5)} = \frac{10}{-4} = -\frac{5}{2}$

The slope of \overleftrightarrow{QT} is $\frac{6 - (-4)}{3 - 7} = \frac{10}{-4} = -\frac{5}{2}$

$RS \parallel QT$ because both lines have the same slope.

The slope of \overleftrightarrow{ST} is $\frac{6 - 0}{3 - (-9)} = \frac{6}{12} = \frac{1}{2}$

The slope of \overleftrightarrow{RQ} is $\frac{-4 - (-10)}{7 - (-5)} = \frac{6}{12} = \frac{1}{2}$

$ST \parallel RQ$ because both lines have the same slope.

Quadrilateral QRST is a parallelogram by definition because both pairs of opposite sides are parallel.

Another way to show that QRST is a parallelogram is to prove that both sides of opposite sides are congruent (using the distance formula to find the lengths of each side).

$$QR = \sqrt{(7 - (-5))^2 + (-4 - (-10))^2} = \sqrt{144 + 36} = \sqrt{180} = 6\sqrt{5}$$

$$ST = \sqrt{(-9 - 3)^2 + (0 - 6)^2} = \sqrt{144 + 36} = \sqrt{180} = 6\sqrt{5}$$

$$QR = ST = 6\sqrt{5}$$

$$RS = \sqrt{(-5 - (-9))^2 + (-10 - 0)^2} = \sqrt{16 + 100} = \sqrt{116} = 2\sqrt{29}$$

$$QT = \sqrt{(7 - 3)^2 + (-4 - 6)^2} = \sqrt{16 + 100} = \sqrt{116} = 2\sqrt{29}$$

$$RS = QT = 2\sqrt{29}$$

Both pairs of opposite sides of quadrilateral QRST are congruent. Therefore, it is a parallelogram.

Extension Questions:

- Use algebra to find the point of intersection of the diagonals of quadrilateral QRST.

To find the point where the diagonals intersect, the equations of lines \overleftrightarrow{RT} and \overleftrightarrow{SQ} must be identified and then used to find the point of intersection.

The slope of \overleftrightarrow{RT} is $\frac{6 - (-10)}{3 - (-5)} = \frac{16}{8} = 2$

The slope of \overleftrightarrow{SQ} is $\frac{0 - (-4)}{-9 - 7} = \frac{4}{-16} = -\frac{1}{4}$

The equation of \overleftrightarrow{RT} is $(y - 6) = 2(x - 3)$ or $y = 2x$

The equation of \overleftrightarrow{SQ} is

$$y = -\frac{1}{4}(x - (-9))$$

$$y = -\frac{1}{4}x - \frac{9}{4}$$

The point where the diagonals intersect can be found by using linear combination.

$$y = 2x$$

$$y = -\frac{1}{4}x - \frac{9}{4}$$

$$2x = -\frac{1}{4}x - \frac{9}{4}$$

$$8x = -1x - 9$$

$$9x = -9$$

$$x = -1$$

$$y = 2x = 2(-1) = -2$$

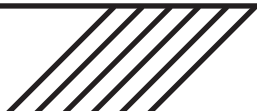
The point of intersection is $(-1, -2)$.

- Use coordinate geometry to prove the diagonals of quadrilateral QRST bisect each other.

The midpoint of \overline{QS} is $\left(\frac{7+(-9)}{2}, \frac{4+0}{2}\right) = (-1, -2)$

The midpoint of \overline{RT} is $\left(\frac{-5+3}{2}, \frac{-10+6}{2}\right) = (-1, -2)$

The midpoints of the segment are the same point as the intersection point. The diagonals bisect each other.



Student Work Sample

Field Test Teacher's Comment:

This was the first performance task I had the students do using a poster. I enjoyed the poster and I feel most of the students did, too. I would like to have done this problem during the quadrilaterals section and will do so next year. One thing I did different this time was to have the students write on the back of their solution guide exactly what to put on their posters for the three criteria we emphasized.

The teacher emphasized three criteria from the Geometry Solution Guide found in the introduction to this book. On the back of one student's solution guide were these notes:

Shows an understanding of the relationships among elements

- Statement showing how the elements are related.
- Can the history teacher understand your steps?

Makes an appropriate and accurate representation of the problem using correctly labeled diagrams

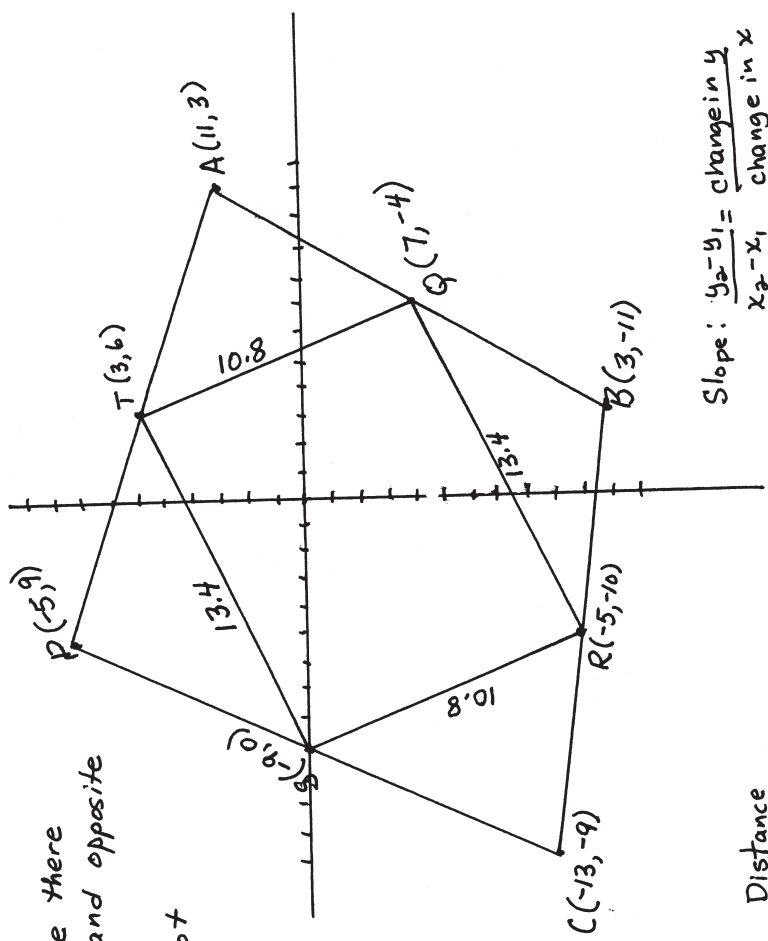
- Drawing the pictures
- Make appropriate markings on the picture

Communicates clear, detailed, and organized solution strategy

- Step by step details that can be followed
- Don't plug in a number without showing why/how
- Must have justification
- Explaining your thinking!!!

A copy of the poster from this student's group appears on the next page.

A parallelogram - because there are two parallel sides and opposite sides are equal.
 The diagonals are also not equal so it is not a rectangle or a square.



Midpoints on graph.

$$RS = \sqrt{(-9-5)^2 + (0-70)^2} = \sqrt{16+100} = \sqrt{116} \approx 10.8$$

$$TQ = \sqrt{(7-3)^2 + (-4-6)^2} = \sqrt{16+100} = \sqrt{116} \approx 10.8$$

Opposite sides are equal.
 $RS = TQ$

$$QR = \sqrt{(-5-7)^2 + (-10-4)^2} = \sqrt{144+36} = \sqrt{180} \approx 13.4$$

$$ST = \sqrt{(-9-3)^2 + (0-6)^2} = \sqrt{144+36} = \sqrt{180} \approx 13.4$$

Opposite sides are equal.
 $QR = ST$

Distance

$$RT = \sqrt{(-5-3)^2 + (-10-6)^2} = \sqrt{64+256} = \sqrt{320} \approx 17.9$$

$$SQ = \sqrt{(-9-7)^2 + (0-4)^2} = \sqrt{256+16} = \sqrt{272} \approx 16.5$$

The diagonals are not equal.

Slope: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$

$$QR: \frac{10-4}{-5-7} = \frac{-6}{-12} = \frac{1}{2} \quad QR \parallel ST \text{ because the slopes are the same.}$$

$$ST: \frac{6-0}{3-9} = \frac{6}{-6} = -1 \quad \text{the slopes are the same.}$$

$$RS: \frac{0-10}{-9-5} = \frac{-10}{-14} = \frac{5}{7} \quad RS \parallel QT \text{ because the slopes are the same.}$$

$$QT: \frac{6-4}{3-7} = \frac{2}{-4} = -\frac{1}{2} \quad \text{slopes are the same.}$$